International Conference

The last 60 years of Mathematical Fluid Mechanics: Longstanding Problems and New Perspectives

In Honor of Professors Robert Finn and Vsevolod Solonnikov

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Abstracts

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Nonlinear parabolic equations on Riemannian manifolds

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We shall present some recent result concerning the local existence and regularity of nonlinear parabolic boundary value problems on Riemannian manifolds. In general, the manifolds can be noncompact and have singularities. Our results cover quasilinear problems in Sobolev space settings as well as fully nonlinear equations in a Hölder space framework. The results are new, even in the standard Euclidean case.

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On the Laplace Parallel Plates Problem

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Laplace considered two parallel plates Π_1 and Π_2 making respective contact angles γ_1 and γ_2 with a fluid, and immersed vertically into an infinite bath of the fluid in a vertical gravity field. He gave persuasive reasoning to show that according as $\Psi(\gamma_1;\gamma_2) \equiv \gamma_1 + \gamma_2 - \pi < 0$ or > 0, the fluid surface between the plates rises or falls unboundedly as the plates are brought together; he seems not to have considered the case of equality. We present a new and more complete description of these behaviors, and establish that if $\Psi(\gamma_1;\gamma_2) = 0$ then the fluid surface tends to an inclined infinitesimal strip attached to the undisturbed level at infinity.

We describe also experiments that verify the predictions in the cases of inequality. The verification proceeds via electrocapillarity phenomena, and in that sense has also independent interest. The height estimates lead to corresponding estimates for the forces between the plates, which relate to earlier ones obtained by Aspley, He and McCuan.

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Singularity formation in geometric flows

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I will consider hypersurfaces in Euclidean space $\mathbb{R}^{n+1}$ evolving by mean curvature flow. If the initial hypersurface is convex, it is well known that the flow shrinks to a point and becomes round after rescaling. If $n > 2$ and the initial hypersurface is 2-convex (in the sense that the sum of the two smallest curvature eigenvalues is positive), Huisken and Sinestrari were able to give a complete classification of the singularities (which are modeled on shrinking cylinders); moreover, they were able to extend the flow by a surgery procedure similar to the one developed by Hamilton and Perelman for the Ricci flow in dimension 3.

In this lecture, I will discuss how this result can be extended to the remaining case $n = 2$; this relies on a sharp estimate for the inscribed radius under mean curvature flow. Moreover, I will describe a similar surgery construction for a fully nonlinear flow, which allows us to treat 2-convex hypersurfaces in Riemannian manifolds. This is joint work with Gerhard Huisken.

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A Floating Cylinder or Ball on An Unbounded Reservoir

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In the study of a circular cylinder horizontally floating on an unbounded reservoir in a gravitational field directed downwards, Bhatnargar and Finn [1] gave the first example with two equilibrium configurations. We give a complete study [2] of the number of equilibria, the floating configurations and their stability. The initial model has a limitation due to the possible intersection of fluid interfaces that is not physically realizable. We show the stable equilibrium point never lies in the intersection region, while the unstable equilibrium configuration may be physically unrealizable. In addition, we illustrate the number of equilibria considering the intersection condition in the $\mathcal{C}$ vs $\mathcal{A}$ plane, where $\mathcal{A}$ and $\mathcal{C}$ are two non-dimensional parameters. Examples with typical contact angles (e.g. $\gamma = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}$ and π) will be given. For a ball floating on an unbounded bath, the non-monotone relation between the height of center h and the wetting angle ϕ_0 makes the problem significantly different than the floating cylinder problem. The case $\gamma = \frac{\pi}{2}$ will be discussed.

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Uniqueness of self-similar solutions obeying the problems of arbitrary discontinuity disintegration for the generalized Hopf equation

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Solutions of the problems of disintegration of an arbitrary discontinuity of the generalized Hopf equation are under analysis. These solutions are constructed from the sequence of non-tipping Riemann waves and shock waves having the stable stationary or non-stationary structure.

In [1, 2], devoted to investigation of the solutions of the Hopf equations with complex nonlinearity, for selection of discontinuities, which have been used for construction of the solution, the request of existence of the stationary structure of the discontinuity has been posed. The structure of discontinuities has been described by the generalized (in the sense of nonlinearity) Korteweg-de Vries-Burgers equation. Appearance of the resent works [3, 4], in which spectral stability of the solutions describing the structure is investigated, makes it possible to include effectively in the notion of the permissible discontinuity the claim of stability of its structure and from this point of view revise before obtained results. We call admissible (i. e. realizable in practice for disintegration of an arbitrary discontinuity) discontinuities with structure, having stability property.

Introduction of the request of stability of the structure in the notion of admissibility of discontinuities results in cutting down the set of admissible discontinuities, described in [1, 2], and eliminate non-uniqueness of the solution of the problem about disintegration of the arbitrary shock, discovered in previous investigations [1]. Furthermore, for construction of the solution of the problem we have used the discontinuities with structure, containing the internal periodic oscillations (non-stationary structures). Variation of the quantities in such discontinuities may not coincide with variation of the quantities in any discontinuities with stationary structure. It has been shown [5] that the solution of the problem of disintegration of the arbitrary discontinuity in this setting always uniquely exists.

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L_2-theory for two incompressible fluids separated by a free interface

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We deal with the problem governing nonstationary motion of two viscous incompressible fluids separated by an unknown interface and contained in a bounded vessel belonging to $\mathbb{R}^3$. It is assumed that the fluids are subjected by mass forces and the capillary forces on the interface.

Existence and uniqueness theorem for the problem in L_2-setting was proved on a finite time interval determined by the norms of the data [1, 2]. The stability of a rest state for the problem without taking surface tension into account was proved when initial velocities were small and mass forces were decaying in an exponential way [3].

Now we continue to treat the problem in the Sobolev-Slobodetskiĭ spaces $W_2^{2+l,l+1/2}$, $l \in (1/2, 1)$, mass force being small and tending to zero as $t \to \infty$ but not necessarily as an exponential function of t. We admit a more general decay of the right-hand side of the Navier–Stokes equation. Moreover, we assume initial velocities to be small and an interface to be close to a sphere at an initial instant. As before in [4], where the study was made in the Hölder spaces, the idea of constructing a generalized energy is used for obtaining an exponential estimate, but now global solvability is first established for a linear problem and only then for the nonlinear one. In addition, Hanzawa's transformation is used to reduce the smoothness of the free interface. It belongs to W_2^{2+l}. The result obtained implies the stability of a rest state, velocity vector field vanishing, the interface tending to a sphere with center different, in general, from the position of the barycenter of the inner drop at the initial instant.

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The global attractor for autonomous quasi-geostrophic equations for the Navier-Stokes equations

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Consider the autonomous quasi-geostrophic equation with fractional dissipation in $\mathbb{R}^2$

$$\theta_t + u \cdot \nabla\theta + (-\Delta)^\alpha \theta = f(x, \theta) \tag{1}$$

in the subcritical case $1/2 < \alpha \leq 1$, with initial condition $\theta(x, 0) = \theta^0$ and given external force $f(x, \theta)$. Here the real scalar function θ is the so-called potential temperature, and the incompressible velocity field $u = (u_1, u_2) = (-\mathcal{R}_2\theta, \mathcal{R}_1\theta)$ is determined from θ via Riesz operators. Our aim is to prove the existence of the compact global attractor $\mathcal{A}$ in the Bessel potential space $H^s(\mathbb{R}^2)$ when $s > 2(1 - \alpha)$.

The construction of the attractor is based on the existence of an absorbing set in $L^2(\mathbb{R}^2)$ and $H^s(\mathbb{R}^2)$ where $s > 2(1 - \alpha)$. A second major step is usually based on compact Sobolev embeddings which unfortunately do not hold for unbounded domains. To circumvent this problem we exploit compact Sobolev embeddings on balls $B_R \subset \mathbb{R}^2$ and uniform smallness estimates of solutions on $\mathbb{R}^2 \setminus B_R$. In the literature the latter estimates are obtained by a damping term $\lambda\theta$, $\lambda < 0$, as part of the right hand side f to guarantee exponential decay estimates. In our approach we exploit a much weaker nonlocal damping term of convolution type $\rho * \theta$ where $\widehat{\rho} < 0$.

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Piecewise constant subsolutions for the incompressible Porous Media Equation

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A recent result by Castro, Córdoba and Faraco in [1] showed the existence of infinitely many weak solutions of the incompressible porous media equations for all Muskat type initial data with H^5-regularity of the interface and in the unstable regime. The main ingredient for the proof was the construction of a suitable subsolution, which was done analogously to the case of the flat interface by Székelyhidi in [2] and leads to a nonlinear evolution equation for the sheet. In this talk I present an alternative proof of this result. The density of the subsolution will be defined to be piecewise constant. In particular, we can choose an arbitrary fine approximation of the continuous density in [1]. The second main step is the observation that we can avoid solving the full nonlinear equation, since a power series ansatz up to order two is sufficient for the construction. Moreover, this method leads to a necessary regularity for the initial interface in the class $W^{4,1}(\mathbb{R}) \cap C^{4,\alpha}(\mathbb{R})$. As already mentioned in [1], we obtain a similar result for Muskat type initial data in the stable regime except of the flat case.

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Free boundary problem of magnetohydrodynamics for two liquids

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We consider the free boundary problem of magnetohydrodynamics in a bounded domain in R^3. It describes the motion of a finite mass of viscous incompressible electrically conducting capillary liquid inside the other viscous incompressible liquid under the action of magnetic field. The interface between the liquids is unknown and subject to capillary forces. We prove local (in time) solvability of the problem in anisotropic Sobolev-Slobodetskii spaces $W_2^{2+l,1+l/2}$, $1/2 < l < 1$. In order to reduce free boundary problem to a problem in a fixed domain, we use the Hanzawa coordinate transform. The linearized problem can be decomposed in two parts: hydrodynamical and magnetic. In the talk we will concentrate on the investigation of the corresponding linear two-phase problem for magnetic field. Existence result for the nonlinear problem is obtained by the successive approximations method.

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Parabolic Equation of Normal Type Connected with 3D Helmholtz System and Its Nonlocal Stabilization

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The talk will be devoted to the normal parabolic equation (NPE) connected with 3D Helmholtz system whose nonlinear term B(v) is orthogonal projection of nonlinear term for Helmholtz system on the ray generated by vector v. Interest to NPE arised in connection with attempts to find approaches to solve problem on non local existence of smooth solution for 3D Navier-Stokes equations.

As it became clear now the studies of NPE has been opened the way to construct the method of nonlocal stabilization by feedback control for 3D Helmholtz as well as for 3D Navier-Stokes equations.

First we describe the structure of dynamical flow corresponding to this NPE (see [1]). After, the non local stabilization problem for NPE by starting control supported on arbitrary fixed subdomain will be formulated. The main steps of solution to this problem will be discussed (see [2]). At last how to apply this result for solution of nonlocal stabilization problem with impulse control for 3D Helmholtz system will be explained.

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Large time behavior of a generalized Oseen evolution operator, with applications to the Navier-Stokes flow past a rotating obstacle

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Consider the motion of a viscous incompressible fluid in a 3D exterior domain D when a rigid body $\mathbb{R}^3 \setminus D$ moves with prescribed time-dependent translational and angular velocities. For the linearized non-autonomous system, L^q-L^r smoothing action near $t = s$ as well as generation of the evolution operator $\{T(t,s)\}_{t \geq s \geq 0}$ was shown by Hansel and Rhandi [1] under reasonable conditions. In this presentation we develop the L^q-L^r decay estimates of the evolution operator $T(t,s)$ as $(t-s) \to \infty$ and then apply them to the Navier-Stokes initial value problem.

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Global Stabilization of Navier-Stokes-Voight Equations and Related Systems by Finiite Number of Feedback Controllers

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We introduce a finite-parameters feedback control algorithm for stabilizing solutions of the Navier-Stokes-Voigt equations and related systems of equations modeling dynamics of viscoelastic fluids. Stabilization of solutions of these problems is established by introducing a feedback control terms that employ parameters, such as, finitely many Fourier modes, finitely many volume elements and finitely many nodal observables and controllers.

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Non-uniqueness of closed non-smooth hypersurfaces with constant anisotropic mean curvature and self-shrinkers of anisotropic mean curvature flow

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We study variational problems for surfaces in the euclidean space with an anisotropic surface energy. An anisotropic surface energy is the integral of an energy density which depends on the surface normal over the considered surface. It was first introduced by Gibbs to model the equilibrium shape of a small crystal. If the energy density is constant one, the anisotropic surface energy is the usual area of the surface. The minimizer of an anisotropic surface energy among all closed surfaces enclosing the same volume is unique (up to translations) and it is called the Wulff shape. Equilibrium surfaces of a given anisotropic surface energy functional for volume-preserving variations are called surfaces with constant anisotropic mean curvature (CAMC surfaces). In general, the Wulff shape and CAMC surfaces are not smooth. If the energy density satisfies the so-called convexity condition, the Wulff shape is a smooth convex surface and closed embedded CAMC surfaces are only homotheties of the Wulff shape. In this talk, we show that if the convexity condition is not satisfied, such a uniqueness result is not always true, and also the uniqueness for self-shrinkers with genus zero for anisotropic mean curvature flow does not hold in general. These concepts and results are naturally generalized to higher dimensions.

On steady Navier–Stokes equations in 2D exterior domains

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The talk is based on some recent joint results obtained with R. Russo (University of Caserta, Italy) and K. Pileckas (Vilnius University).

In the paper [2] the existence theorem was proved for steady NS-system in 2D exterior domains under Amick symmetry conditions (see [1]). In our talk we discuss further development of this result, i.e., the existence theorems and asymptotic properties for this system (without symmetry assumptions).

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Some properties of solutions of the Navier-Stokes Equations with various types of boundary conditions

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We solve systems of the Navier-Stokes equations with various types of boundary conditions. We study qualitative properties of solutions of these systems. Some properties of these solutions, e.g. local in time existence of strong solutions, are presented. Further, we deal with perturbations of initial velocities of strong solutions of these systems. We prove that corresponding solutions are also strong for sufficiently small perturbations in some norms.

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Steady flow around a two-dimensional rotating body

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I will discuss some new results concerning a steady flow of an incompressible viscous fluid, which is at rest at spatial infinity, around a two-dimensional body rotating with constant nonzero angular velocity $a \neq 0$. In the linearized case, namely the Stokes system in a rotating frame of reference, every solution decays as $|x| \to \infty$, and the asymptotic profile is known. However, the constants in the corresponding a apriori estimates exhibit a singular behavior as $a \to 0$. I will present some new estimates in which this singular behavior is quantified, and discuss how to employ them in the analysis of the fully non-linear problem.

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The Behavior of Capillary Surfaces along Edges

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The behavior of nonparametric capillary surfaces in vertical cylinders has been studied for centuries and yet new discoveries continue to be made. For capillary surfaces in "tubes" which might not be vertical cylinders and for sessile drops on a horizontal plane (with variable contact angle), the situation is even more interesting.

As in [1, 2, 4, 5, 6], we will discuss some of the effects of a lack of smoothness in the geometry of the container or, for a sessile drop, a discontinuity of the contact angle. The talk will focus on some of the results in [1]-[6] but will (hopefully) explore other aspects of this topic (e.g. sessile drops, tubes).

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Boundary Behavior of a Capillary Surface with Extremal Contact Angle

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Let $f(x, y)$ describe a surface of constant mean curvature in Ω which has contact angle 0 along a relatively open piece Σ' of the boundary arc $\partial\Omega$. Under some assumptions on p and Σ', if the boundary trace is Lipschitz continuous in a relatively open piece of the boundary arc, then based on results in [1], around this piece of boundary arc we shall show that the solution is Hölder continuous of order $1/2$ up to the boundary.

Furthermore, we consider the moon domain $\widehat{\Omega}$ in $\mathbb{R}^2$, which is bounded by two circular arcs Σ^- and Σ^+, of the respective radii $1/2$ and R, $1/2 < R < 1$, and is characterized by the requirement $2|\widehat{\Omega}| = |\Sigma^+| - |\Sigma^-|$. Consider the vertical cylinder Z over $\partial\widehat{\Omega}$, with the sides Z^+ and Z^- of Z over the circular arcs Σ^+ and Σ^-. The moon surface $z = \widehat{f}(x, y)$ is defined over $\widehat{\Omega}$ which is of constant mean curvature 1 and meets the sides Z^+ and Z^- in the respective angles 0 and π. We shall study the behavior of the moon surface in the neighborhood of the corner points V_1 and V_2 where Σ^+ meets Σ^- and show that the radial limit either exists in each direction or does not exist in any direction. If the radial limit exists in each direction, then the radial limit will be shown to be strictly decreasing as the direction of approach becomes further away from the tangent direction of Σ^+ at V_1 or V_2.

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Mathematical Study on Gaseous Stars

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Evolution of self-gravitating gaseous stars is governed by the Euler-Poisson equations:

$$\frac{\partial \rho}{\partial t} + \sum_{j=1}^{3} \frac{\partial}{\partial x^j}(\rho v^j) = 0,$$

$$\rho\Big(\frac{\partial v^j}{\partial t} + \sum_{k=1}^{3} v^k \frac{\partial v^j}{\partial x^k}\Big) + \frac{\partial P}{\partial x^j} = -\rho \frac{\partial \Phi}{\partial x^j}, \quad j = 1, 2, 3,$$

$$\sum_{j=1}^{3} \Big(\frac{\partial}{\partial x^j}\Big)^2 \Phi = 4\pi \mathsf{G} \rho.$$

Here $t \geq 0, x = (x^1, x^2, x^3) \in \mathbb{R}^3$. ρ is the density, $v = (v^1, v^2, v^3)$ is the velocity field. The pressure P is supposed to be a given function of ρ as $P = \mathsf{A}\rho^\gamma$, A, γ being positive constants such that $1 < \gamma \leq 2$. Φ is the gravitational potential and G is a positive constant.

Mathematical analysis of this system of equations gives rise to difficulties, when we consider compactly supported density distributions. Actually the equations of continuity and motions vanish on the vacuum region. Existence of time-local solutions to the initial value problem for compactly supported density was first established in [1] as an application of the theory of quasi-linear symmetric hyperbolic systems. But there was a weak point: this study required $\rho^{(\gamma-1)/2} \in H^3(\mathbb{R}^3) \subset C^1$, but equilibria given by the Lane-Emden equation have finite radii R, provided that $6/5 < \gamma \leq 2$, and behave as $\rho \sim \text{Const.}(R - r)^{1/(\gamma-1)}$ as $r \to R - 0$ so that $\rho^{(\gamma-1)/2} \sim \text{Const.}(R - r)^{1/2} \notin C^1$. It was suggested that this vacuum boundary affair requires treatment as a free boundary problem. See [2, p. S223].

After 30 years this point was overcome, at least for spherically symmetric solutions, by [3] as an application of the Nash-Moser theorem formulated by R. S. Hamilton. Time evolutions of the form

$$\rho(t, r) = C(t)(R_F(t) - r)^{\frac{1}{\gamma-1}}(1 + O(R_F(t) - r))$$

near the vacuum boundary $r = R_F(t)$ were established. But a restriction that $\gamma/(\gamma - 1)$ be an integer is required. If we use the formulation of the Nash-Moser theorem by J. T. Schwartz, we can treat the case in which $\gamma/(\gamma - 1)$ is not an integer but $1 < \gamma < 54/53$, while $P = \mathsf{A}\rho^\gamma(1 + O(\rho^{\gamma-1}))$ as $\rho \to +0$ but $P \neq \mathsf{A}\rho^\gamma$ exactly. See [4]. Independently Juhi Jang studied the same problem under wider range of γ. See [5]. She does not use the Nash-Moser theorem but uses the Hardy inequality.

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These results can be extended to the problem in the general theory of relativity, in which the Einstein-Euler equations govern the evolution of gaseous stars. See [6], [4].

On the other hand, although the structure of spherically symmetric equilibria is well-known, but it is not the case for axially symmetric stationary solutions of the Euler-Poisson equations which give mathematical models of uniformly rotating gaseous stars. We have studied them in [7] and constructed solutions under the assumption that the angular velocity is small and $6/5 < \gamma \leq 3/2$. The evolution problem near this uniformly rotating axisymmetric stationary solution, given by the "distorted Lane-Emden function" so called after [7], is still now under construction and open.

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Navier-Stokes flow past a rigid body: attainability of steady solutions as limits of unsteady weak solutions, starting and landing cases

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Consider the Navier-Stokes flow in 3-dimensional exterior domains, where a rigid body is translating with prescribed translational velocity $-h(t)u_\infty$ with constant vector $u_\infty \in \mathbb{R}^3 - \{0\}$. Finn raised the question whether his steady solutions are attainable as limits for $t \to \infty$ of unsteady solutions starting from motionless state when $h(t) = 1$ after some finite time and $h(0) = 0$ (starting problem). This was affirmatively solved by Galdi, Heywood and Shibata [1] for small u_∞. We study some generalized situation in which unsteady solutions start from large motions being in $L(3, \infty)$. We then conclude that the steady solutions for small u_∞ are still attainable as limits of evolution of those fluid motions which are found as a sort of weak solutions. The opposite situation, in which $h(t) = 0$ after some finite time and $h(0) = 1$ (landing problem), is also discussed. In this latter case, the rest state is attainable no matter how large u_∞ is.

The text of the abstract of paper [2].

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Pseudo-equilibrium grain boundaries

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This work is based on a model for grain boundary evolution in which the interfaces between grains evolve under mean curvature flow and the grain - vapor interfaces evolve under surface diffusion flow. The physical systems which inspired this model exhibit both growth and extinction of individual grains. The model exhibits these behaviors as well, though the fundamental geometric mechanism according to which grain growth and/or extinction occurs remains unclear.

Pseudo-equilibrium configurations are those satisfying some subcollection of the conditions necessary for equilibrium. For complicated variational problems, especially those in which there are very few equilibrium configurations or for which necessary and sufficient conditions for equilibrium are not known, classifying and studying families of pseudo-equilibria can sometimes lead to insights concerning the geometry of the underlying problem. We describe how this approach may be applied to some simple idealized grain boundaries with symmetry.

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The Muskat problem and related topics

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The present talk is devoted to the Muskat problem, which describes the joint motion of two immiscible incompressible liquids (for example, oil and water) in an absolutely rigid solid skeleton. This physical process is very important from practical point of view and any valuable mathematical model of the process must have real applications. Unfortunately, the oil industry is still using only the hydrodynamic simulator of the oil reservoirs ”Eclipse”, which based upon the constantly criticized by applied mathematicians very old Backley-Leverett model.

The Muskat problem consists of two Darcy laws for two different liquids divided by unknown (free) boundary. It is well known that Darcy is a homogenization of the Stokes equations. But a numerical homogenization of the joint motion of two immiscible viscous liquids in periodic structure governed by Stokes equations doesn't result the Muskat problem [1]. The arising free boundary problem is the subject of the first part of our talk. A global in time classical solvability is proved.

Everything is changed if we consider the motion of liquids in elastic skeleton at the pore level. For the corresponding mathematical model at the pore level and for the homogenized model there exist unique global in time weak solutions [1]. In the second part of the talk we prove that the free boundary problem at the pore level has a unique global in time classical solution.

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A numerical approach to the second boundary value problem for the curvature operator

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The boundary value problem is formulated in [1]

$$\nabla \cdot \frac{\nabla \varphi}{\sqrt{1+|\nabla\varphi|^2}} - \beta\varphi = f(x'), \quad x' = (x_1, x_2) \in \Omega \subset \mathbb{R}^2,$$

$$\frac{\partial \varphi}{\partial \nu} = 0 \quad \text{on } \partial\Omega.$$

Here Ω is a bounded 2-D domain with smooth boundary, β is a positive constant, ν is the unit outward normal to $\partial\Omega$, $\nabla u = (\partial u/\partial x_1, \partial u/\partial x_2)$.

$$A[\varphi] \equiv \nabla \cdot \frac{\nabla \varphi}{\sqrt{1+|\nabla\varphi|^2}} \quad \text{is the double mean curvature of the surface}$$

$\Gamma = \{x_3 = \varphi(x'),\ x' \in \Omega\}$. The problem is solved numerically for $\Omega = \{0 < x_1 < a, 0 < x_2 < b\}$ by successive approximation method. The operator A is decomposed into linear and nonlinear parts. $A[\varphi] = \Delta\varphi + R[\varphi]$. The initial approximation φ_0 is the solution to the Neumann problem

$$\Delta\varphi_0 - \beta\varphi_0 = f, \quad x' \in \Omega, \quad \frac{\partial \varphi_0}{\partial \nu} = 0 \quad \text{on } \partial\Omega.$$

Subsequent approximations φ_n are determined as solutions to the Neumann problems

$$\Delta\varphi_n - \beta\varphi_n = f - R[\varphi_{n-1}], \quad x' \in \Omega, \quad \frac{\partial \varphi_n}{\partial \nu} = 0 \quad \text{on } \partial\Omega, \quad n = 1, 2, ...$$

The solution to the Neumann boundary value problem is constructed as a Fourier series.

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Decay estimates of solutions for the Navier-Stokes-Korteweg system in $\mathbb{R}^N$

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In this talk, we consider the Navier-Stokes-Korteweg system in $\mathbb{R}^N$:

$$\begin{cases} \partial_t \rho + \operatorname{div}(\rho \mathbf{u}) = 0 & \text{in } \mathbb{R}^N,\ t \in (0,T), \\ \rho\,(\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u}) - \operatorname{Div}(\mathbf{S} + \mathbf{K}) + \nabla P(\rho) = 0 & \text{in } \mathbb{R}^N,\ t \in (0,T), \\ (\rho, \mathbf{u})|_{t=0} = (\rho_* + \rho_0, \mathbf{u}_0) & \text{in } \mathbb{R}^N. \end{cases}$$

Here ρ, $\mathbf{u}$ and P denote the density, velocity and pressure of fluid, respectively. The viscous stress tensor $\mathbf{S}$ and the Korteweg stress tensor $\mathbf{K}$ are $\mathbf{S} = 2\mu\mathbf{D}(\mathbf{u}) + (\nu - \mu)\operatorname{div}\mathbf{u}\mathbf{I}$, and $\mathbf{K} = (\kappa\rho\Delta\rho + \kappa/2|\nabla\rho|^2)\mathbf{I} - \kappa\nabla\rho\otimes\nabla\rho$ with $\mathbf{I}$ the $N\times N$ identity matrix and $\mathbf{D}(\mathbf{u}) = \{\nabla\mathbf{u} + (\nabla\mathbf{u})^T\}/2$, μ, ν are viscosity coefficients, and κ is a capillary coefficient. ρ_* is a positive constant. This system was first introduce by Korteweg D. J.[1] and Van der Waals J. D.[2].

It is shown that the system admits a unique, global strong solution for small initial data in the L_p in time and L_q in space setting. For the purpose, the main tools are the maximal L_p-L_q regularities and L_p-L_q decay properties to the linearized equations.

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Qualitative properties of solutions to elliptic and parabolic equations with divergence-free lower-order coefficients

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We consider uniformly elliptic and uniformly parabolic equations of divergence type:

$$\mathcal{L}u \equiv -D_i\big(a_{ij}(x)D_ju\big) + b_i(x)D_iu = 0; \qquad (\mathbf{DE})$$

$$\mathcal{M}u \equiv \partial_t u - D_i\big(a_{ij}(x;t)D_ju\big) + b_i(x;t)D_iu = 0 \qquad (\mathbf{DP})$$

with additional structure condition

$$\operatorname{div}(b_i) \le 0 \quad \text{in the sense of distributions.} \qquad (1)$$

The equations with the lower-order coefficients satisfying this structure condition arise in some applications, in particular, in hydrodynamics.

We deal with classical properties of solutions, namely, strong maximum principle, Hölder estimates, the Harnack inequality and the Liouville Theorem. We show that under condition (1) the assumptions on (b_i) which ensure these properties can be considerably weakened in the scale of Morrey spaces.

This talk is based on a joint paper with Nina N. Ural'tseva, see [1]. Author was supported by RFBR grant 15-01-07650.

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Application of the relative entropy inequality in moving domains

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The relative energy inequality was introduced by Dafermos [1] and in the fluid context introduced by Germain [5]. Deriving the relative energy inequality for sufficient smooth test functions and proving the weak-strong uniqueness it gives us very powerful and elegant tool for the purpose of measuring the stability of a solution compared to another which has a better regularity. This method was developed by E. Feireisl, A. Novotny and co-workers in the framework of singular limits problems (see for example [2], [3] and [4]). The aim of the lecture is the application ofthe weak-strong uniqueness to the case of the compressible Navier-Stokes system in the time-dependent domain and to the case of motion of rigid body in a bounded domain filled by incompressible fluid. For more details see [6] and [7].

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Martingales and mean curvature

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Surfaces of prescribed (including constant and zero) mean curvature are natural objects of interest when discussing equilibrium states of fluids. Such surfaces are also frequently amenable to methods of stochastic analysis, largely due to the fact that the mean curvature vector appears naturally in the semimartingale decomposition in the ambient space of Brownian motion on the surface. In this talk, we discuss this connection, especially applications to classical minimal surfaces, as developed in [2], [1], and [4] (see also [3] for a survey targeted at geometers).

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On the structure of the set of stationary solutions to the equations of motion of a class of generalized Newtonian fluids

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We deal with the steady–state equations of motion of the generalized Newtonian fluid in a bounded domain $\Omega \subset \mathbb{R}^N$, when $N = 2$ or $N = 3$. Applying the tools of nonlinear analysis (Smale's theorem, properties of Fredholm operators, etc.), we show that if the dynamic stress tensor has the so called 2–structure then the solution set is finite and the solutions are C^1–functions of the external volume force $\mathbf{f}$ for generic $\mathbf{f}$. We also derive a series of properties of related operators in the case of a more general (p, δ)–structure, show that the solution set is compact if $p > 3N/(N+2)$ and explain why the same method as in the case $p = 2$ cannot be applied in the case of general p.

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Instability of compressible Poiseuille flow and traveling waves

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The evolutional stability of the Poiseuille flow of compressible viscous fluids is obtained in [1] under the smallness condition of both Reynold number and Mach number. If the Mach number is not small, a linear instability occurs for much smaller Reynolds number compared with the linear critical Reynolds number of incompressible Poiseuille flow [2].

The instability gives the bifurcation of periodic traveling waves for the nonlinear system [3].

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Study of unsteady flow near a critical point

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Many works (see the classical results in [1]) have been dedicated to the problem of the flow of a viscous incompressible liquid near a critical point, i.e., the flow around a wall perpendicular to the flow. Its exact stationary solution was obtained by Hiementz in 1911. We investigate the problem of unsteady flow of a viscous incompressible liquid near a critical point on a planar boundary. The problem reduces to the solving of the following initial-boundary value problem for integrodifferential equation

$$q_t + q^2 - q_y \int_0^y q(s,t)ds - 1 = q_{yy}, \quad y > 0, \quad t > 0;$$

$$q(y,0) = q_0(y), \quad t \geq 0; \quad q(0,t) = 0, \quad t > 0; \quad q \to 1, \quad y \to \infty, \quad t > 0.$$

A theorem on the existence and uniqueness of its solution in Holder classes of functions on an arbitrary time interval with natural restrictions on the initial function is proved. The main difficulties in the proof of the solvability theorem is the quadratic nonlinearity, and also the unboundedness of the region and the presence of a divergent integral in front of the first derivative with respect to the first derivative with respect to the spatial variable. Qualitative properties of the solution are investigated. Results of a numerical analysis demonstrate the possibility of disappearance after a finite time of a counterflow zone existing at the initial time in the case of a negative pressure gradient at the hard plane. In the case when the pressure gradient is a periodic function, a periodic regime of motion as well as breakdown of the solution after a finite time is possible [2].

Self-similar unsteady solutions describing plane and axisymmetric viscous incompressible fluid flows in the vicinity of a stagnation point located on the solid boundary are constructed. Fluid flow regimes from infinity to the stagnation point and from it are considered. The problem under consideration depends on two parameters. As a result, two different flow regimes are obtained [3].

Similar questions arise in the study of the problems of motion of the Maxwell medium [4].

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Nonlinear waves in the Maxwell continuum

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Wave processes in incompressible visco-elastic Maxwell medium are considered. They are described by system of quasi-linear equations of composite type. A class of solutions is selected, for which this system gets disintegrated on hyperbolic and elliptic parts. Depending on the choice of objective derivative in equation of state, there are possible layered flows with either strong or weak discontinues.

If we choose Jaumann derivative for objective derivative, then equations of layered motions are identically equal to inviscid gas dynamics equations with non-convex equation of state. This complicates solution to Riemann problem about breakup of discontinuity. In case of upper or lower convective derivative equations of layered motions are linear and coincide with equations of string of variable density.

On the basis of effectively one-dimensional models with upper or lower convective derivatives in rheological relation, non-stationary motions near the critical point are studied. Transition to Lagrange coordinates allows us to formulate initial-boundary value problem for symmetrical in sense of Friedrichs hyperbolic system. This system belongs to class of weakly non-linear hyperbolic systems, studied by Yanenko. It doesnt admit strong discontinues in motions, while solutions with weak discontinues are possible.

In general three-dimensional case characteristics of system of equations of the 10th order are calculated. They are comprised by two complex characteristics, fourfold contact characteristics and four wave characteristics. Under the choice of Jaumann derivative and lower convective derivative, there exist two different speeds of nonlinear shear waves propagation, while in case of upper convective derivative in rheological relation they coincide.

Relaxation time τ enters as coefficient at higher derivative in the behavior law, which is the cause for appearing pulsations in non-stationary problems. This is well illustrated on example of linearized problem, for which the problems for finding fields of velocity, stress and pressure are separated. Here an unexpected result is fast stabilization of pressure field at $\tau \to 0$ on the background of high-frequency oscillations of velocity vector.

As for stationary problem, there was build asymptotic decomposition with respect to small relaxation time τ, which does not contain functions of boundary layer type. Typical example is a problem on stationary flow near the critical point. Its solution when relaxation time approaches zero transits to well-known Hiemenz solution.

The case of large relaxation times is interesting for the fact that limit model at $\tau \to \infty$ admits Lie pseudo-group significantly wider than the initial one. On the basis of invariant solutions of limit system there were built approximate solutions to initial system in form of the power series in $1/\tau$ powers.

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Stable mild Navier-Stokes solutions

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In the talk we will consider a variant of Kato-Fujita's and Giga-Miyakawa's approximation schemes: By iterative solution of linear singular Volterra integral equations, on any compact time interval J, again we find the existence of a unique mild Navier-Stokes solution in any smoothly bounded domain $\Omega \subset \mathbb{R}^n$, $n \geq 2$, under smallness conditions. Moreover we get the stability of each (possibly large) mild solution inside a scale of Banach spaces which are imbedded in some $C^0(J, L^r(\Omega))$, $1 < r < \infty$.

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General Inverse Problems with Linear Functional Constraints and Some Applications

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In this work, in collaboration with Simo Correia, we consider an abstract initial value problem for an equation with linear operators A and B in Hilbertian frameworks of the type

$$Bu'(t) + Au(t) = q(t)\zeta(t) + f(t) \tag{1}$$

subjected to linear constraints, defining continuous time controls,

$$\langle z(t), u(t)\rangle = F(t), t \in (0, T), \tag{2}$$

where the unknown pair (u, q) is uniquely determined by the given data f, z, ζ and F.

As examples of applications, when B is the identity, we consider the Poiseuille flow in a open valley model with prescribed flux and pointwise controls in the domain for higher order problems. The case of a pseudoparabolic partial differential equation arising in seepage of fluids through a fissured rock and another model on the heat equation with dynamic and diffusion conditions on the boundary complete our applications.

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Solution of boundary value problems for surfaces of prescribed mean curvature $H(x, y, z)$ via the continuity method

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Dedicated to Professor Dr. Robert Finn in very hight respect

Since more than a century minimal surfaces and capillarity phenomena as well as surfaces of prescribed mean curvature are in the center of mathematical interest (see [5], [2], [1]). When we consider surfaces of prescribed mean curvature H with a one-to-one orthogonal projection onto a plane, we have to study the nonparametric H-surface equation. E. Heinz proposed in his paper [4] to introduce conformal parameters into these surfaces and study F. Rellich's H-surface-system.

The essential step to receive a compactness result for this class of surfaces consists of the area estimate by R. Finn [3]. He originally established this method for equations of minimal surface type. This estimate has been generalized to H-surfaces by E. Heinz. Via the Courant-Lebesgue lemma we can establish a modulus of continuity and thus linearize the nonlinear elliptic H-surface system by the gradient estimate of E. Heinz (see [7] Chapter 12).

Now the H-surfaces with a one-to-one central projection onto a plane lead to an interesting elliptic differential equation, which has been invented by T. Radó [6] in the case $H = 0$. We establish the uniqueness of the Dirichlet problem for this H-surface equation in central projection. Moreover, we develop an estimate for the maximal deviation of large H-surfaces from their boundary values, resembling an inequality by J. Serrin [11]. Then we provide a Bernstein-type result for the case $H = 0$ and classify the entire solutions of the minimal surface equation in central projection.

Now we solve the Dirichlet problem for $H = 0$ by a variational method. In the central part [9] we solve the Dirichlet problem for nonvanishing H via a nonlinear continuity method. Here we use a differential equation for the normal to these surfaces [8] in order to preserve the property of one-to-one central projection. Furthermore, we construct large H-surfaces (compare the result [12] by M. Struwe) bounding extreme contours by an approximation. Finally, we solve the Dirichlet problem on strictly convex domains for the nonparametric H-surface equation in central projection with the aid of a maximum principle [10] for H-surfaces in the unit cone.

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Asymptotic Problems in Capillarity

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Since the earliest interest in equilibrium fluid confguraions, asymptotic problems have attracted attention, [2]. We will survey rigorous asymptotic results for axisymmetric problems, liquid inside or outside a narrow vertical cylindrical tube, and for non-axisymmetric problems in which the domain has a corner or cusp and the fluid interface is unbounded, [1], [3], [4] and [5]. Many challenging problems remain.

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A regularity criterion for the Navier-Stokes equations in terms of one item of the velocity gradient

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We study the conditional regularity of solutions to the Navier-Stokes equations in the whole three-dimensional space. In our presented criterion we impose additional assumptions only on one item of the velocity gradient, $\partial_1 u_3$, where $u = (u_1, u_2, u_3)$ is the velocity. For the proof of our result we use conveniently the anisotropic Lebesque spaces and a suitable version of the Troisi inequality - see [1].

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Some remarks about the Helmholtz-decomposition in domains with boundary singularities

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Decompositions of vector fields into a solenoidal part and a gradient field, $u = u_0 + \nabla p$, play an essential role in the theory of Navier-Stokes equations. They are constructed by solving special weak Laplace-problems for the function p. If we choose a weak Neumann condition for p this decomposition is usually called the Helmholtz-decomposition. By combining well known results for elliptic boundary value problems with duality arguments and garnishing this menu with some technical tricks it is possible to construct Helmholtz decompositions for domains with various types of boundary singularities - including a control over the asymptotic behaviour of p.

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Mathematical justification of basic equations of nonlinear acoustics

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Broadly speaking, the branch of fluid mechanics known as acoustics can be defined as the study of irrotational compressible flow.

If the signal strengths involved are relatively weak and the propagation distances of interest are relatively short, then the inherently nonlinear system of equations that describe sound propagation can, often to rather good accuracy, be approximated by linear PDEs.

In contrast, when the problems of interest involve 'finite-amplitude' acoustic signals and/or extreme propagation distances, simple linear theory usually proves to be inadequate. This occurs because the effects of nonlinearity, being both present in the fundamental equations of fluid flow and cumulative in nature, rapidly become felt over time and distance. As such, it would appear that we are compelled, when confronted with such problems, to set aside our simple linear models in favor of there *fully nonlinear*, and thus more challenging, counterparts. There is, however, another possibility; the so-called *weakly nonlinear* modeling approach. By this we mean the derivation of approximate equations of motion, which are based on the 'small, but finite-amplitude' (*i.e.*, small Mach number) assumption, from the irrotational Euler and Navier–Stokes–Fourier systems that, while relatively tractable from the mathematical standpoint, still capture the salient nonlinear phenomena exhibited by compressible flows.

In this communication, we are concerned with some basic concepts and modern investigations of nonlinear and thermoviscous phenomena in acoustic fields in fluids. The contents of my talk are as follows:

- To derive basic equations of sound mode from irrotational viscous compressible Navier–Stokes equations through a weak approximation; new basic equation similar to the equations due to Kuznetsov [1] and Blackstock [2] is derived from the real approximation standpoint;
- Global-in-time existence of a small solution to viscous compressible Navier–Stokes equations with specific pressure and temperature;
- Global-in-time existence of a small solution to the new equation mentioned above;
- Justification of the approximation.

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On the classification and asymptotic behavior of the symmetric capillary surfaces

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We consider the symmetric solutions to the Young-Laplace equation, and its extensions past vertical points. We provide a classification of all symmetric solutions using certain families of parameters. This classification produces a unified approach to fluid interfaces in capillary tubes, sessile and pendent drops, liquid bridges, as well as exterior and annular capillary surfaces. The generating curves for the symmetric solutions have asymptotes for large arclengths, and the behavior of these asymptotes is analyzed.

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Quasilinear obstacle problems in bad domains

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In this talk I will focus the attention on obstacle problems involving p-Laplace type operators in domains with fractal boundary, corresponding pre-fractals problems, smoothness properties, asymptotic behavior, FEM-approximation and the error estimates.

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Stability of Delaunay Surface Solutions to Capillary Problems

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The relation between stability and local energy minimality will be outlined. This will be specialized to capillary surfaces, and in particular to Delaunay surfaces. The same Delaunay surface can solve different capillary problems with different boundary conditions. Stability of the surface in these different problems will be explored. In particular, the stability of a liquid bridge between parallel planes will be related to the stability of a liquid bridge between solid balls. This is a summary of research spanning several decades, starting from [1] and continuing until [2].

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